According to Algorithms in Everyday Mathematics, for each operation the teacher materials identify one of the alternative algorithms as the *focus algorithm*, which is to provide a back-up for students who do not otherwise achieve proficiency and to provide a common basis for further work. However, students are encouraged to use whatever method they prefer. The focus algorithms are "partial sums" for addition, "trade first" for subtraction, "partial products" for multiplication, and "partial quotients" for division. In the subsections that follow, in each case the focus algorithm is described first.

## Addition

For addition, Everyday Mathematics offers four methods. The "partial sums" method and the "column addition" method are already present in the 3rd grade student reference book and remain the two methods of choice in 4th grade. The "fast method" (traditional) and the "opposite change rule" make their appearance in the 5th grade student reference book.

### Partial Sums Method

The Partial Sums method (the Everyday Math focus algorithm for addition) is a two-stage process. In the first stage one looks at each column (working left to right) and adds up the place-values represented by the digits in that column. In the second stage those partial sums are added together. In the first example at right the process is applied to 148 + 67 + 266. The Everyday Mathematics student reference does not recommend a specific algorithm for the addition problem in the second stage. Frequently the second stage problem will be "easy" in that it can be done one column at a time without any carries, as is the case in the first example. Perhaps the pupil is expected to iterate the Partial Sums method in cases where the second stage addition problem involves carrying.

The reader may want to explore the Partial Sums algorithm in a case where the second stage and even the logical third stage involve carrying. Try 148 + 855 or, for an example that is a bit harder for mental arithmetic, see the second problem at right, 678 + 67 + 266. The mathematically inclined reader will note that the Partial Sums method (iterated so long as carrying is required) terminates, because every application of the basic step will reduce, from the right, the range of columns for which carrying is required.

```
     148
678
   +  67
+  67
   + 266
+ 266
     ---
---
 ->  300         -
>  800
   + 160
+ 190
   +  21
+  21
     ---
---
     481         -
>  900
+ 110
+   1
----
                ->
etc
```

### Column Addition Method

The Column Addition method is also a two-stage process. It is

```
1| 4| 8
```

recommended to write the addends in wide columns separated by vertical lines. In the first stage one adds up the digits one column at a time and writes down the result (which may be a two-digit number) in each column. In the second stage, working right to left, one does the carries to obtain the result in the standard base-ten representation.

```
     +  | 6| 7
     + 2| 6| 6
       --|--|--
        3|16|21
 ->     3|18| 1
 ->     4| 8| 1
 ->     481
```

## A Fast Method (traditional)

This is the traditional method of right to left addition. Everyday Mathematics has adopted the early learner's variant in which the carries are explicitly written down, above the addends. (The more advanced traditional pupil would do the carries mentally.)

```
 (12 )
  148
+  67
+ 266
  ---
  481
```

## Opposite Change Rule

The Opposite Change rule is not a standard algorithm; it is only a tool to help simplify addition of a pair of numbers. The rule says that the sum of two numbers is not changed if one subtracts a number from one addend and adds the same number to the other addend. This can be used to make an addition problem easier by making one of the two addends end in one or more zeroes. The process is not uniquely specified, obviously. In the problem at right, 185 + 266, we might have used the rule to change the expression into 190 + 261, or into 181 + 270, but then the resulting addition problem still requires carrying. In that case I suppose that we can iterate, continuing to simplify the same term, but the Everyday Mathematics student text assumes that a single application of the rule will turn the problem into one that the pupil can do without further difficulty.

```
    185   ->
200
  + 266        +
251
    ---          -
--

451
```

A nastier case for the Opposite Change rule would be 185 + 263. If we apply the rule to change the first term into 180 or 190, or to change the second term into 260 or 270, then the resulting problem still involves carrying (besides which, we may have to do a subtraction with borrowing in making the change to the other term). We can change the 185 to 200 but then have to do a subtraction with borrowing on the other term. The cleanest application in this case may be the one shown at right, but this just goes to illustrate that the Opposite Change rule is truly not a standard algorithm.

```
          ??
    185   ->
205
  + 263        +
243
    ---          -
--

448
```

## Subtraction

For subtraction, Everyday Mathematics offers five methods. The "trade first method", the "left to right subtraction method", and the "counting up method" are already present in the 3rd grade student reference book, and the "partial differences method" and the "same change rule" make their first appearance in the 4th grade student reference. Notice that the traditional right to left subtraction method is not part of the Everyday Mathematics curriculum.

### Trade First

Closest to the traditional standard is a method that Everyday Mathematics calls "trade first". It is the Everyday Math focus algorithm for subtraction. It is a two stage process, first working right to left to do all the borrowing (recording the intermediate results above the top number) and then a second pass, in any order, doing the subtractions. The intermediate results are two-digit numbers, so one needs to use wide columns, and it is recommended to separate the columns with clear vertical lines. In the example at right, 325 - 58, we first recognize that the ones column needs borrowing, so we replace 2*10+5 by 1*10+15. Then we recognize that the tens column also needs borrowing, so we replace 3*100+1*10 by 2*100+11*10. Then we do the subtraction in each column.

```
  2|11|
   | 1|15
  3| 2| 5
-  | 5| 8
 --|--|--
  2| 6| 7
```

### Left to Right Subtraction

The second standard method of EM is left to right subtraction, the way one might well do the problem mentally, but carried out with paper and pencil. Here "left to right" refers to the decomposition of the second number. In the example at right, 325-58, the 58 is decomposed as 50+8. The individual subtractions are done mentally.

```
  325
- 50
  ---
  275
-   8
  ---
  267
```

### Counting Up

The third standard method in EM is the "counting up method". It is carried out in two stages. In the first pass we count up from the smaller to the larger number, first by ones, then tens, and so on, and then the odd remainder, and then in a second pass we add up the addends. The example at right displays the process on our familiar example, 325 - 58.

```
      58        and
then
   +   2
      60
2
   +  40
+  40
     100
+ 200
   + 200
+  25
```

```
    300
---
  +  25
267
    325
```

**Partial Differences**

The fourth standard method of EM, the "partial differences" method, is again a two-stage method. We operate first on each column individually, keeping track of the sign if a borrow would be needed, and then we combine the results using mental arithmetic. With this method, for 325 - 58 the second stage involves the mental arithmetic 300 - 30 - 3 = 267.

```
  325
 - 58
  ---
 +300
 - 30
 -  3
  ---
  267
```

**Same Change Rule**

The fifth method is the "same change" rule. It is based on the recognition that a subtraction problem is easier if the smaller number ends in one or more zeroes. We may bring that about without changing the answer by changing both terms by the same amount. So, 325 - 58 may be changed into 327 - 60, which we do mentally to obtain our 267. If the second problem is still difficult then we might apply the same change rule again, but this is not discussed in the Everyday Math student reference book.

```
    325   ->
327
   - 58        -
60
    ---         --
-

267
```

## Multiplication

For multiplication, Everyday Mathematics offers four methods. The "partial products method" and the "lattice multiplication method" are already present in the 3rd grade student reference book, and these remain the methods of choice in 4th and 5th grade. In the 6th grade Everyday Math student reference book the "short method" (a version of the traditional algorithm) and the "Egyptian multiplication method" make their appearance.

**Partial Products Method**

The Partial Products Method is the Everyday Math focus algorithm for multiplication. In the Partial Products Method one takes the base-ten decomposition of each factor and forms the products of all pairs of terms. Then these partial products are added together. The student text does not recommend any particular addition algorithm for this second stage. In the

```
                    83
                    27
                  ----
  80*20   ->      1600
  80* 7   ->       560
   3*20   ->        60
   3* 7   ->        21
                  ----
```

example at right I've assumed traditional addition with carries done mentally, but an Everyday Mathematics pupil may well do that addition problem by the Partial Sums or the Column Addition method.

```
2241
```

Observe that the number of terms in the addition problem is the product of the numbers of digits in the factors. The reader may want to explore Partial Products on the example 121 * 121 and compare with the traditional method, or try 128 * 128 for an example that is less skewed towards the traditional method.

### Lattice Method

The lattice method employs a grid of squares. One factor is written along the top, left to right, and the other factor is written along the right edge, top to bottom. In the example at right the factors are 83 and 27. Each square of the grid defined by the two factors is divided by a diagonal. The digits of the factors are multiplied pairwise and the two-digit result written down in the corresponding square in the manner shown. The result of the multiplication is then obtained by addition down the diagonals. The reference book suggests to use here the "fast method" for addition, working right to left and writing the carries in an appropriate box. In the example at right I used instead the column addition method of Everyday Mathematics, although I don't know if the Everyday Math teachers' guide would approve of that. In any case, the reader can as well visualize fast addition down the diagonals after the products have been filled in. I'm a bit puzzled by Everyday Math's preference for the use of fast addition here, since it is not the preferred method in Everyday Mathematics in the cleaner context of adding up a set of numbers written above one another.

```
      8   3
    +---+---+
    |1 /|0 /|
    | / | / | 2
  1 |/ 6|/ 6|
    +---+---+
    |5 /|2 /|
    | / | / | 7
 11 |/ 6|/ 1|
    +---+---+
     14   1

   1,11,14,1
-> 1,12,4,1
-> 2,2,4,1
 = 2241
```

### A Short Method

The "Short Method" is Everyday Math's version of the traditional multiplication algorithm. In the short method only the second factor is decomposed. Each digit of the second number is interpreted according to its place value, and the partial product of that term with the first number is written down as shown. Then those partial products are added together. The student reference text does not describe how the pupil is expected to do the single-digit times multi-digit

```
              83
              27
            ----
 7*83 ->     581
20*83 ->    1660
            ----
            2241
```

multiplication problem. The manner of the addition step is also left open.

The Everyday Math student reference books up to and including 6th grade do not discuss the traditional right to left method for paper and pencil multiplication of a single-digit and a multi-digit number, nor is it expected that students will be able to do this mentally working left to right. The "Short Method" as just described makes its appearance only in the 6th grade student reference book. It seems a fair guess that students may carry out this algorithm by doing the individual single-digit times multi-digit products in a separate area on the paper with use of the partial products or the lattice multiplication method. They may also do the addition step separately with use of the Everyday Mathematics focus algorithm of Partial Sums.

**Egyption Method**

The Egyptian Method of multiplication is a three-stage process. Some might understand it as the base-2 traditional algorithm carried out in base ten. In the first stage one forms two columns, one for each factor. The entries in both columns are formed by repeated doubling. The first column starts with a 1, and doubling is done until one reaches the largest power of two not exceeding the first factor. The second column starts with the second factor, and repeatedly doubles as often as was done in the first column. In the second stage, working bottom to top, one identifies the subset of entries in the first column that adds up to the first factor, and one crosses out the other entries. (In effect, one determines the base-2 representation of the first factor.) In the third stage one adds up the corresponding entries in the second column. The example at right assumes that the addition is done using the fast traditional algorithm with carries done mentally. It might be fairer, however, to assume that the student would use the Everyday Mathematics focus algorithm of partial sums for the addition step.

| | | |
|---|---|---|
| | 83 | * 27 |
| + | 1 | 27 |
| + | 2 | 54 |
| | ~~4~~ | ~~108~~ |
| | ~~8~~ | ~~216~~ |
| + | 16 | 432 |
| | ~~32~~ | ~~864~~ |
| + | 64 | 1728 |
| | -- | -- -- |
| | 83 | 2241 |

## Division

For division, Everyday Mathematics offers two methods: the "partial quotients method" and the "column division method". The traditional method of long division is not taught.

**Partial Quotients Method**

The Partial Quotients Method, the Everyday Mathematics focus algorithm for division, might be described as successive approximation. It is suggested that a pupil will find it helpful to prepare first a table of some easy multiples of the divisor; say twice and five times the divisor. Then we work up towards the answer from below. In the example at right, 1220 divided by 16, we may have made a note first that 2*16=32 and 5*16=80. Then we work up towards 1220. 50*16=800 subtract from 1220, leaves 420; 20*16=320; etc..

```
      ------  |
  16 ) 1220   |
      - 800   |
50
        ----  |
         420  |
       - 320  |
20
         ---  |
         100  |
       -  80  |
5
         ---  |
          20  |
       -  16  |
1
          --  |  -
-
           4  |
76

  ans: 76 R4
```

**Column Division Method**

The column division method appears to be the traditional method, but implemented in a rather verbose manner, and presented in a way that constitutes a viable student algorithm only when the divisor is a single-digit number. The Everyday Mathematics 5th and 6th grade student reference books present it via a visualization as money sharing. In the example at right we divide 1220 by 7. We have one $1000 note, which can not be shared by 7 people. We change it to 10 $100 notes, giving us 12 $100 notes in all. With 7 people each gets 1 such note, and we mark the 1 above the dividend. This removes 7 $100 notes, leaving us with 5, which we convert into 50 $10 notes, giving us 52 such notes in all. With 7 people each can get 7 such notes, which we mark above the dividend. This removes 49 $10 notes, leaving us with 3, which we convert into 30 $1 notes, giving us indeed 30 $1 notes in all. With 7 people each can get 4 such notes, which we mark above the dividend. This removes 28 such notes, leaving us with 2. That 2 is our remainder, and the integer result of the division is read above the line.

```
       0 | 1 | 7
| 4
    -----+---+---
+--
  7 )  1 | 2 | 2
| 0
     - 0
     ----+---
       1 |
  ->     |12
         - 7
         ----+---
           5 |
       ->    |52
             -49
             ----
+---
               3
|
           ->
|30

-28

---

2
  ans: 174 R2
```